UND-HEP-00-BIG13
hep-ph/0012336

A Few Aspects of Heavy Quark Expansion

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To appear in the Proceedings of the UK Phenomenology Workshop on
Heavy Flavour and CP Violation
Durham, UK, 17–22 September 2000

Abstract

Two topics in heavy quark expansion are discussed. The heavy quark potential in perturbation theory is reviewed in connection to the problem of the heavy quark mass. The nontrivial reason behind the failure of the "potential subtracted" mass in higher orders is elucidated. The heavy quark sum rules are the second subject. The physics behind the new exact sum rules is described and a simple quantum mechanical derivation is given. The question of saturation of sum rules is discussed. A comment on the nonstandard possibility which would affect analysis of $\mathrm{BR}_{\mathrm{sl}}(B)$ vs. n_c is made.

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1 Introduction

Heavy quark symmetry and the heavy quark expansion have played an important role in understanding weak decays of heavy flavors. Recent years witnessed significant success in quantifying strong nonperturbative dynamics in a number of practically important problems via application of Wilson Operator Product Expansion (OPE). The advances were in theoretical understanding of strong dynamics of heavy quarks, as well as in practical applications to phenomenology of beauty decays. The discussion of both aspects can be found in the recent review [1]. It also included updates on such questions of general interest as the current theoretical uncertainties in extracting the KM parameters $|V_{cb}|$ and $|V_{ub}|$, our knowledge of the heavy quark masses and other basic parameters of the heavy quark theory, aspects of local quark-hadron duality relevant to the quest for $|V_{cb}|$ and $|V_{ub}|$. The interested reader can find there references to the original publications.

In this contribution I address a few selected topics which have not been spelled in much detail in the literature. Even though some are of more theoretical nature, I try to place emphasis on the general physical features understandable without presenting much of the technicalities. I discuss in some detail the notion of the heavy quark potential in QCD and its limitations, in particular as viewed through its relation to the heavy quark mass. The origin of the subtleties is explained which led to a failure in defining the low-scale running heavy quark mass in higher orders using the usual interquark potential.

Another topic is the heavy quark sum rules, where the significant part is dedicated to new exact spin sum rules; applications to phenomenology are discussed. I conclude with the brief comment on charm counting in B decays.

2 Heavy quark potential and the heavy quark mass

Strictly speaking, the heavy quark potential appears in a somewhat different situation compared to usual B decays, namely where heavy quark and antiquark (with small relative velocities) are present simultaneously. It attracted recently renewed attention in connection with the pair production of $t\bar{t}$ and $b\bar{b}$ near threshold, and through its connection with the problem of heavy quark mass. The review of applications for the $Q\bar{Q}$ system can be found, for example, in Ref. [2]. Here we give a more pedagogical discussion of the underlying problems.

A closer look at the notion of the heavy quark potential in QCD reveals certain subtleties. It turns out that a literal analogue of potential interaction does not exist in QCD.

The original notion of the potential refers to the interaction of infinitely heavy (static), or completely nonrelativistic heavy particles, which is *instantaneous*. The most familiar example is the electromagnetic interaction of heavy charges. The

Hamiltonian of such a potential system is given by

$$\mathcal{H} = \sum_i \frac{(i\vec{\partial}\,)^2}{2m_i} + V\left(\vec{r}_i - \vec{r}_j\right) \;, \tag{1}$$

where m_i are masses of particles and the potential V is a function of their instant coordinates. Taking the limit $m_i \to \infty$ (at fixed $\vec{r}_i$, which corresponds to semiclassically high excitation numbers of the quantum system in Eq. (1)) eliminates quantum uncertainties in the coordinates and allows to measure the potential directly as the position-dependent energy of the infinitely slowly moving collection of particles. This is well-known for QED where the potential of the charges q_1 and q_2 (in units of the electron charge) is given by

$$V^{\rm QED}(R) = \alpha_{\rm em}(0)\, \frac{q_1 q_2}{R} \;. \tag{2}$$

This expression is exact in the absence of light charged particles; known quantum corrections appear only if other matter fields are not much heavier than the scale $1/R$.

The definition of the similar quantity in QCD turns out more tricky, since color sources are gauge-dependent. The color of the individual heavy quark remains fully quantum in nature and changes through interaction with gluons. In contrast to usual coordinates, the limit $m_Q \to \infty$ does not make color variables describing the state of the heavy quark semiclassical.

To avoid this problem, the heavy quark potential is usually defined via the vacuum expectation value of stretched Wilson loops,

$$V(R) = -\lim_{T\to\infty} \frac{1}{T} \ln \left\langle {\rm Tr}\, \mathcal{P} \exp\left(i \oint_{C(R,T)} A_\mu {\rm d}x_\mu \right) \right\rangle \;, \tag{3}$$

where the rectangular contour $C(R,T)$ spans distances R and T in the space and time directions, respectively. (It is usually assumed to be in Euclidean space as in Eq. (3).) The gauge field is taken in the color representation of the heavy quark (fundamental representation for actual quarks). This definition is intuitively clear, since such Wilson lines describe propagation of infinitely heavy quarks. Moreover, such Wilson loops are readily computed in the U(1) gauge theory (free QED) and, of course, reproduce Eq. (2) (with $q_1 = -q_2$). In QCD the perturbative expansion of $V(R)$ has been computed through order α_s^3.[3] Usually the potential in the momentum representation is considered,

$$V(\vec{q}\,) = \int {\rm d}^3\vec{R}\; V(\vec{R})\; {\rm e}^{-i\vec{q}\vec{R}} = -\frac{4}{3}\frac{4\pi\alpha_s}{\vec{q}^{\,2}} \left\{ 1 + \left(\frac{31}{3} - \frac{10}{9} n_f\right) \frac{\alpha_s}{4\pi} + c_3 \left(\frac{\alpha_s}{4\pi}\right)^2 + \ldots \right\}. \tag{4}$$

Taking the definition (3) for $V(R)$, one faces a number of questions. What type of processes does $V(R)$ incorporate and what are its properties, say, in the

perturbative expansion? Can it be used, say, in the Schrödinger equation similar to usual quantum mechanics or QED? It turns out that both questions are nontrivial and interrelated.

There are important peculiarities in the thus defined static interaction in the $Q\bar{Q}$ system; they were first analyzed by Appelquist *et al.* already in the late 1970's [4]. Due to the gluon self-interaction, the potential contains, in higher orders, the H-type diagrams shown in Figs. 1. Here the dashed line denotes the Coulomb quanta (they mediate instantaneous interaction in the physical Coulomb gauge), while the wavy lines are used for the transverse gluons. These diagrams propagate in time the transverse gluons. This means that at this level the problem ceases to be a two-body one, and includes more full-fledged dynamical degrees of freedom.

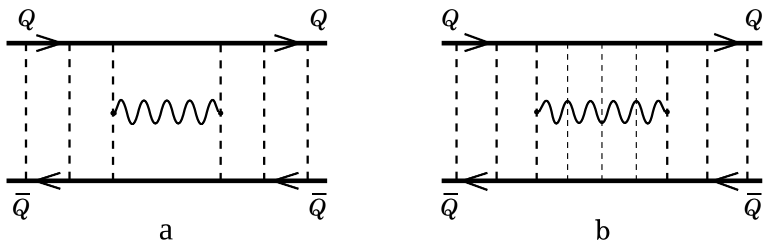

Figure 1: Examples of diagrams for heavy quark potential in QCD. Dashed lines are instantaneous Coulomb exchanges, transverse gluons propagating in time are shown by wavy lines. a) The convergent H-diagrams with the transverse gluon as a rung. b) Adding more Coulomb exchanges inside the "H" leads to infrared divergence in perturbation theory.

Moreover, while the first H-diagram Fig. 1a appearing in order α_s^3 safely converges at the rung gluon momenta $\sim 1/R$, including additional Coulomb exchanges, as in Fig. 1b leads to an infrared divergence. With one exchange it is logarithmic; the formal degree of the infrared divergence increases with adding extra Coulomb quanta between the emission and absorption of the transverse gluon. Physics behind this infrared behavior was discussed in Ref. [4]: emission of the soft transverse gluon changes the overall color of the $Q\bar{Q}$ pair and, therefore, modifies the interaction energy between them. The energy shift associated with the exchange of the transverse gluon depends nonanalytically on the energy denominator (in the language of noncovariant time-ordered perturbation theory), since the gluon can be arbitrarily soft. Formally expanding the exact result in α_s includes expanding in this change of the Coulomb energy proportional to α_s/R. Therefore, one obtains increasing infrared singularities. These arguments suggested that resummation of the Coulomb exchanges in Fig. 1b would render these diagrams finite. However, the effective infrared cut-off is of the order of α_s/R and, thus, the potential is not *perturbatively* infrared finite containing terms $\sim 1/R \cdot \ln \alpha_s$ starting at the order $\mathcal{O}(\alpha_s^4)$.

This purely perturbative analysis shows that there is no direct analogue of the potential between heavy quarks in QCD. The $Q\bar{Q}$ system incorporating all gluon

interactions is not a two-body system but includes actual propagation of gluons with energy small compared to $1/R$. The interaction, then, cannot be universally described by an instantaneous potential and is intrinsically nonlocal in time. Thus, the prescription (3) based on Wilson loops in QCD does not yield the heavy quark potential in its conventional understanding. Theoretical treatment of the $Q\bar{Q}$ system has to account for these peculiarities of QCD. Examples of the technique used here can be found in Ref. [2] and other papers referred to there.

In spite of all subtleties mentioned above, the heavy quark potential V defined via Wilson loops, Eq. (3), is an observable quantity (at least in the perturbative regime $R \ll 1/\Lambda_{\rm QCD}$), up to an overall additive constant appearing in the renormalization of the straight Wilson line. In particular, it is gauge-invariant in perturbation theory. It is therefore tempting to use it to quantify various strong interaction effects, including low-energy contribution to the mass of the heavy quark.

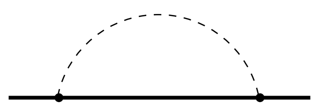

Figure 2: Self-energy diagram yielding the classical Coulomb shift in the mass of a nonrelativistic particle.

It is well known that in classical electrodynamics the self-energy of a charged particle is given by the potential at the origin, $\frac{1}{2}e^2V(0)$. The same holds in quantum electrodynamics in respect to the effect of quanta with momenta much smaller than the source mass (or inverse radius, for the composite particle), as is illustrated by the diagram Fig. 2 yielding

$$\delta m \simeq e^2 \int \frac{d^3\vec{k}}{4\pi^2}\frac{\alpha}{\vec{k}^2} = \frac{1}{2}\, e^2\, V_{\rm em}(0)\ . \tag{5}$$

Here e is charge and $V_{\rm em}$ is the potential between the same-sign charges, hence it is positive. In QCD the perturbative diagrams for both m_Q and $V(\vec{q})$ are more complicated. Yet, the similar relation holds in order $\mathcal{O}(\alpha_s^2)$ as well. This is most simply seen in the Coulomb gauge where all the effect at this order reduces to dressing the propagator of the Coulomb quanta, i.e. using the running $\alpha_s(\vec{q}^{\,2})$. (An alternative discussion can be found in Ref. [5].) There is a general argument [1] showing that such a relation holds to all orders in perturbation theory: the infrared contribution to $V(0)$,

$$V_{\rm IR}(0) = \int \frac{{\rm d}^3\vec{q}}{(2\pi)^3}\, V_{\rm IR}(\vec{q})\ . \tag{6}$$

equals to minus twice the same contribution to the mass of a static color source.

The general structure of the perturbative diagrams for heavy quark mass and $V(0)$ indeed is similar. To establish the correspondence, one can cut the $\bar{Q}$ line in the diagram for the potential at some place and turn the new external legs around.

Integration over momentum $\vec{q}$ corresponds to closing the original external Q and $\bar{Q}$ legs, as exemplified by Fig. 3. This correspondence often works out in a nontrivial way. For example, in the potential we should discard the "reducible" diagrams which can be cut across only Q and $\bar{Q}$ lines, Fig. 4.a. The corresponding contributions in δm_Q would include rainbow diagrams like in Fig. 4b. The latter, however, vanish in the Coulomb gauge, which is self-manifest in the coordinate representation. The Coulomb propagator is instantaneous, $\delta(t_1-t_2)/|\vec{x}-\vec{y}\,|$ while the heavy quark propagator is retarded and includes $\theta(t_1-t_2)$. In diagrams like Fig. 4b we have $\tau_2>\tau_1$ and therefore they vanish. The exception is one-loop diagram which does not vanish since in the integral over ω the large semicircle at $\omega\to\infty$ yields the finite contribution; this is equivalent to the prescription $\theta(0)=\frac{1}{2}$.

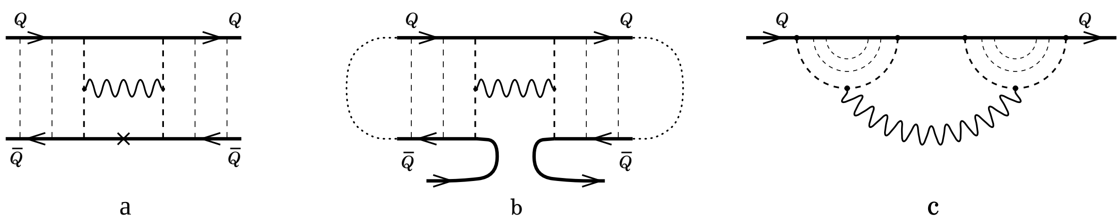

Figure 3: Correspondence between the diagrams for potential $V(0)$ and for the heavy quark mass m_Q. The dotted lines denote closing the external quark lines and integrating over $\vec{q}$.

The general argument goes as follows. Let us imagine we were able to introduce in some way an ensemble of gauge field configurations where the modes with momenta much larger than a certain scale μ are practically absent. This field-theoretic system would not need regularization, and everything can be expressed in terms of the bare parameters, including the bare quark mass $m_Q^{(0)}$. At $R\to 0$ the Wilson loop will approach its free value N_c thus yielding $V(0)=0$. This is clear on physical grounds: $Q\bar{Q}$ form a dipole with the infinitesimal dipole moment, and its interaction with any soft gluon field vanishes as R goes to 0. It is important at this point that our gauge ensemble explicitly includes only soft modes. Otherwise, as in full QCD, the modes with $|\vec{k}\,|\sim 1/R$ generate growing attractive potential at arbitrary small R.

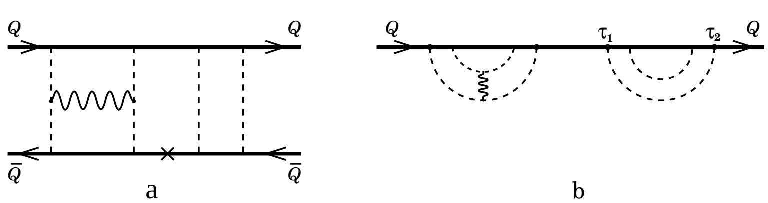

Figure 4: Reducible $\bar{Q}Q$-only diagrams are not included in $V(R)$ being iterations of the potential interaction. Similar diagrams in m_Q vanish due to instantaneous nature of the Coulomb interaction.

Next, we note that $V(R)$ is traditionally determined up to a constant. In per-

turbation theory (in four dimensions) the potential is actually defined as

$$U(R) = V(R) - V(\infty)\,. \tag{7}$$

We assign U to this "standard" potential to distinguish it from $V(R)$ which has a precise meaning in a finite theory. While $U(R)$ by definition vanishes at $R \to \infty$, $V(\infty)$ does not and reflects nontrivial interaction with the gluon field. In the momentum representation $V(\vec{q}\,)$ explicitly contains the term $V(\infty)\,\delta^3(\vec{q}\,)$, which is discarded in standard perturbative computations. Since at $R\to\infty$ the $\bar{Q}$ and Q lines are well separated, their interaction must vanish as $1/R$, (at least, in perturbation theory) and the value of the Wilson loop is given by the mass renormalization of each static source,

$$V(\infty) = 2\left(m_Q^{\rm ren} - m_Q^{(0)}\right) = 2\delta m_Q\ . \tag{8}$$

Therefore, we have for the "ordinary" potential

$$U(0) = \int \frac{{\rm d}^3\vec{q}}{(2\pi)^3}\, V_{\rm reg}(\vec{q}\,) = -2\delta m_Q\ . \tag{9}$$

Here $V_{\rm reg}$ is the regular part of $V(\vec{q}\,)$ (computed in a usual way) not containing self-energy diagrams yielding $\delta^3(\vec{q}\,)$. Thus, we have the stated relation between $V_{\rm IR}(0)$ and $\delta_{\rm IR} m_Q$.

Relying on relation (9), one can try to perturbatively define a certain running heavy quark mass[1] which is free from the leading renormalon uncertainty $\sim \Lambda_{\rm QCD}$ [6]. One defines

$$m_Q^{\rm PS}(\mu) = m_Q^{\rm pole} + \tfrac{1}{2}\int_{|\vec{q}\,|<\mu} \tfrac{{\rm d}^3\vec{q}}{(2\pi)^3}\, V_{\rm reg}(\vec{q}\,) =\ m_Q^{\rm pole} + \tfrac{1}{\pi}\int_0^\infty {\rm d}R\, V(R)\Big[\tfrac{\sin\mu R}{R} - \mu\cos\mu R\Big]\,, \tag{10}$$

where $V(\vec{q}\,)$ is computed to a certain order in perturbation theory with no explicit cut-off, and the pole mass is taken to the same order in α_s. The mass $m_Q^{\rm PS}(\mu)$ is known as the "potential-subtracted" mass [5]. As is clear from the preceding derivation, this mass would have a meaning of the rest-frame energy of the heavy quark, but not the mass determining the kinetic energy, which is generally different once a cutoff is introduced (i.e., m_0 rather than m_2 in the notations of Refs. [7, 1]).

The *ansatz* (10) is supposed to remove from the pole mass all infrared contributions originating from the scale much below the cutoff μ. As we saw, this happens to one and two loops. Unfortunately, this is not true starting order α_s^3.

The problem becomes self-manifest at order α_s^4 where infrared-singular contributions of Fig. 1b emerge in $V(\vec{q}\,)$ which behave like $\frac{\alpha_s^4}{\vec{q}^{\,2}}\ln\frac{\vec{q}^{\,2}}{\epsilon}$, with ϵ being an infrared cutoff in the "rung" gluon momentum. On the other hand, the corresponding contributions are absent from m_Q since the Coulomb exchanges are instantaneous.

[1] This idea was discussed by myself in 1996 and later, independently, advocated by M. Beneke [5].

This apparent contradiction does not mean that the relation (9) is violated. As explained in Ref. [1], the subtlety resides in the necessity to introduce the cut-off on the gluon momenta to compute $V_{\rm IR}$. Here we consider this problem from the different perspective.

Let us take a look at diagram Fig. 3a. It does contribute to $V(0)$ computed via the simple-minded prescription in Eq. (10). At the same time the diagram Fig. 3c vanishes simply since the one-loop $\bar{Q}Qg$ vertex is zero for transverse gluon. The *ansatz* (10) treats the diagrams in Figs. 3a and 3c differently: the integrals run over all momenta of the Coulomb quanta in the latter, but there is a nontrivial cut at momenta of order μ for potential in Fig. 3a. Our general proof actually states that the amplitudes of emitting the transverse gluon must vanish once the incoming Q and $\bar{Q}$ lines are contracted performing the integration over $\vec{q}$. This is the cancellation of the diagrams in Fig. 5, which are subdiagrams to the whole potential $V(0)$. Clearly, the cancellation holds only if the cuts in all diagrams are the same, which is not the case in the *ansatz* (10).

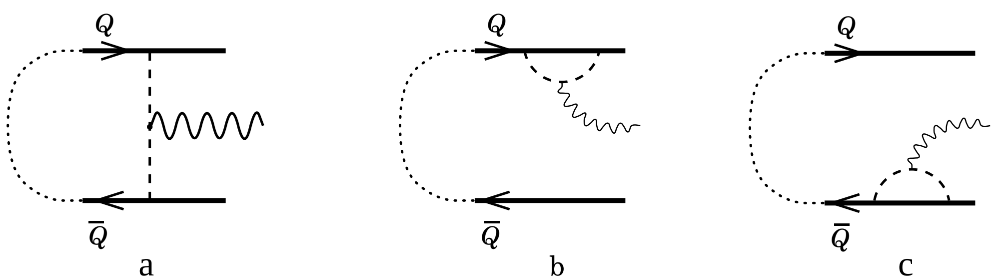

Figure 5: Diagrams for interaction of the transverse gluon which cancel upon integration over $\vec{q}$ in the absence of hard Coulomb quanta. The cancellation, however does not occur for potential in Eq. (10).

The above consideration reveals the problem of the definition of the "potential subtracted" mass at the technical level. In fact, there is a deeper reason behind its failure, which is instructive. The purpose of the additional term in Eq. (10) is to remove the infrared contributions from the pole mass. If the gluons in the diagrams were all soft with momenta not exceeding certain scale ϵ, the induced potential in the momentum representation would vanish completely above some scale $\mu \gg \epsilon$. Then $\int_{|\vec{q}|<\mu} \mathrm{d}^3\vec{q}\; V_\epsilon(\vec{q})$ would comprise this contribution completely and correctly, regardless of any technical details. Why then does *ansatz* (10) fail in practice?

The reason lies in presence of both soft ($\vec{q} \lesssim \epsilon$) and hard ($\vec{q} \sim \mu$) gluons in the potential, and in lack of necessary "factorization" of these scales in $V(\vec{q})$. Let us single out the soft transverse gluon with momentum $\vec{k} \sim \epsilon$. If it affected the full potential only at $|\vec{q}| \lesssim \epsilon$, everything would work fine. However, as exemplified by the H-diagrams in Fig. 1, this is *not* the case. Presence of hard gluons propagates the contribution of the soft gluon to all Fourier components of $V(\vec{q})$. The impact of hard gluons on processes with soft transverse gluons is not limited to only renormalizing their bare interactions, but also introduces new nontrivial 'dipole' matrix

elements between the heavy quark states, which, for example, do not vanish when the momentum $\vec{k}$ goes to zero.

The definition of the "kinetic" running heavy quark mass based on the small velocity sum rules is protected against such problems by the operator product expansion (OPE). The effect of any soft gluon on the moments of the SV structure functions is given by the corresponding heavy quark operators whether or not hard gluons are present. Likewise, the OPE bars soft physics from being transferred to large energy – the corresponding effects are suppressed by at least third power of energy [8] and are described by the OPE.

The lesson one can draw from the analysis of the heavy quark potential/mass problem is that extreme care must be exercised in treating the infrared safe observables for which the OPE does not apply.

3 Heavy quark sum rules

An important class of constraints on the hadronic parameters determining the properties of heavy flavor hadrons follow from the heavy quark sum rules, among which the small velocity (SV) sum rules in the static limit $m_Q \to \infty$ play a special role. These sum rules are

$$\varrho^2 - \frac{1}{4} = 2\sum_m |\tau_{3/2}^{(m)}|^2 + \sum_n |\tau_{1/2}^{(n)}|^2 \,, \tag{11}$$

$$\frac{1}{2} = 2\sum_m |\tau_{3/2}^{(m)}|^2 - 2\sum_n |\tau_{1/2}^{(n)}|^2 \,, \tag{12}$$

$$\frac{\overline{\Lambda}}{2} = 2\sum_m \epsilon_m |\tau_{3/2}^{(m)}|^2 + \sum_n \epsilon_n |\tau_{1/2}^{(n)}|^2 \,, \tag{13}$$

$$\overline{\Sigma} = 2\sum_m \epsilon_m |\tau_{3/2}^{(m)}|^2 - 2\sum_n \epsilon_n |\tau_{1/2}^{(n)}|^2 \,, \tag{14}$$

$$\frac{\mu_\pi^2}{3} = 2\sum_m \epsilon_m^2 |\tau_{3/2}^{(m)}|^2 + \sum_n \epsilon_n^2 |\tau_{1/2}^{(n)}|^2 \,, \tag{15}$$

$$\frac{\mu_G^2}{3} = 2\sum_m \epsilon_m^2 |\tau_{3/2}^{(m)}|^2 - 2\sum_n \epsilon_n^2 |\tau_{1/2}^{(n)}|^2 \,, \tag{16}$$

$$\frac{\rho_D^3}{3} = 2\sum_m \epsilon_m^3 |\tau_{3/2}^{(m)}|^2 + \sum_n \epsilon_n^3 |\tau_{1/2}^{(n)}|^2 \,, \tag{17}$$

$$-\frac{\rho_{LS}^3}{3} = 2\sum_m \epsilon_m^3 |\tau_{3/2}^{(m)}|^2 - 2\sum_n \epsilon_n^3 |\tau_{1/2}^{(n)}|^2 \,, \tag{18}$$

a sequence which, in principle, can be continued further. Here ϵ_k is the excitation energy of the k-th intermediate state ("P-wave states" in the quark-model language),

$$\epsilon_k = M_{H_Q^{(k)}} - M_{P_Q} \;,$$

while the functions $\tau_{3/2}^{(m)}$ and $\tau_{1/2}^{(n)}$ describe the transition amplitudes of the ground state B meson to these intermediate states. We follow the notations of Ref. [9],

$$\frac{1}{2M_{H_Q}}\langle H_Q^{(1/2)}|A_\mu|P_Q\rangle = -\tau_{1/2}\,(v_1-v_2)_\mu\,, \tag{19}$$

and

$$\frac{1}{2M_{H_Q}}\langle H_Q^{(3/2)}|A_\mu|P_Q\rangle = -\frac{1}{\sqrt{2}}\,i\,\tau_{3/2}\,\epsilon_{\mu\alpha\beta\gamma}\,\varepsilon^{*\alpha}\,v_2^\beta\,v_1^\gamma\,, \tag{20}$$

where 1/2 and 3/2 mark the quantum numbers of the light cloud in the intermediate states, $j^\pi = 1/2^+$ and $3/2^+$, respectively, and A_μ is the axial current. Furthermore, the slope parameter ϱ^2 of the Isgur-Wise function is defined as

$$\frac{1}{2M_{P_Q}}\langle P_Q(\vec{v})|\bar{Q}\gamma_0 Q|P_Q\rangle = 1-\varrho^2\,\frac{\vec{v}^{\,2}}{2}+\mathcal{O}(\vec{v}^{\,4})\,. \tag{21}$$

Equation (11) is known as the Bjorken sum rule [10]. Superconvergent sum rules (12) and (14) are new [1, 11]. Equation (13) was obtained by Voloshin [12]. The expression for kinetic expectation value μ_π^2 is the BGSUV sum rule [13]. The next one for chromomagnetic operator was derived in Ref. [14], as well as Eq. (18). The last two sum rules are obtained along the same lines. The sum rule for the Darwin term ρ_D^3 was first presented in Ref. [15]. We have introduced the new parameter of the heavy quark theory $\overline{\Sigma}$; it is the small-velocity elastic transition matrix element between the states with explicit spin of light degrees of freedom: for the ground-state vector mesons such as B^*

$$\frac{1}{2M_{B^*}}\langle H_Q(\vec{v},\varepsilon')|\bar{Q}iD_jQ(0)|H_Q(0,\varepsilon)\rangle = -\frac{\overline{\Lambda}}{2}v_j(\vec{\varepsilon}^{\,\prime *}\vec{\varepsilon}\,) - \frac{\overline{\Sigma}}{2}\Big\{\varepsilon_j'^*(\vec{\varepsilon}\,\vec{v}\,)-(\vec{\varepsilon}^{\,\prime *}\vec{v}\,)\varepsilon_j\Big\}+\mathcal{O}\Big(\vec{v}^{\,2}\Big). \tag{22}$$

The exact magnitude of the nonperturbative hadronic parameter $\overline{\Sigma}$ is not known at the moment. Comparing the sum rules (12), (14) with the sum rule (16) for the chromomagnetic expectation value μ_G^2 we expect $\overline{\Sigma}$ to be about 0.25 GeV. In the nonrelativistic system $\overline{\Sigma}$ is given by the product of the light mass and the orbital momentum, and would vanish for the ground-state mesons.

The sum in the r.h.s. of Eq. (14) was previously considered in the recent paper [16]. It was shown to determine one of the subleading $B \to D^*$ formfactors near zero recoil.

The new spin sum rules (12) and (14) are convergent and are not renormalized by perturbative effects. This distinguishes them from all other heavy quark sum rules. The unified derivation of the sum rules in the field-theoretic OPE is described in detail in the dedicated papers [17], with their quantum mechanical interpretation elucidated. A more pedagogical derivation can be found in recent reviews [14, 1]. The new sum rules were derived in Ref. [11] applying the OPE to the nonforward SV scattering amplitude off the heavy quark. Below we will give an alternative quantum-mechanical derivation. Let us, however, first discuss physics behind the

first sum rule (12) which is the relation for the total angular momentum of light cloud in B meson.

At first sight sum rule (12) which is independent of the strong dynamics looks surprising. In the quark models the $\frac{1}{2}$- and $\frac{3}{2}$-states are differentiated only by spin-orbital interaction. The latter naively can be taken arbitrarily small if the light quark in the meson is nonrelativistic. To resolve this apparent paradox we note that in the nonrelativistic case τ^2 are large scaling like inverse square of the typical velocity of the light quark, $\tau^2 \sim 1/\vec{v}_{\rm sp}^{\,2}$. The relativistic spin-orbital effects must appear at the relative level $\sim \vec{v}_{\rm sp}^{\,2}$ because spin ceases to commute with momentum to this accuracy due to Thomas precession. The latter phenomenon lies behind the sum rule (12). This connection will be elucidated below. The above relativistic corrections lead to the terms of order 1 in the first sum rules Eqs. (11), (12).

To understand the connection of the sum rule (12) with spin, let us consider a small velocity weak transition amplitude M for an elementary particle A. Let it be the scalar vertex, that is, mediated by the scalar current J, $M = \langle A(v')|J(0)|A(v)\rangle$. We consider it in a frame moving with small velocity $\vec{v}$. To simplify consideration, we can assume that the change in velocity $\Delta\vec{v}$ is even smaller: $|\vec{v}\,'-\vec{v}\,| = |\Delta\vec{v}\,| \ll |\vec{v}\,| \ll 1$. For scalar particle A the transition amplitude is described by a single formfactor, so that in this kinematics

$$\langle A(\vec{v}\,')|J(0)|A(\vec{v}\,)\rangle \simeq {\rm const}(1 - a\,(\delta\vec{v}\vec{v}\,)\,)\;. \tag{23}$$

If acceleration proceeds in the direction transverse to the velocity, this amplitude does not depend on the absolute velocity of the particle.

For spin-$\frac{1}{2}$ particle the scalar amplitude likewise is described by a single formfactor, however the nonrelativistic amplitudes has the new structure:

$$\langle A(\vec{v}\,')|J(0)|A(\vec{v}\,)\rangle \simeq {\rm const}\,(\varphi^\dagger\varphi - \tfrac{1}{4}\,i[\delta\vec{v}\times\vec{v}\,]\cdot\varphi^\dagger\vec{\sigma}\varphi - a\,(\delta\vec{v}\vec{v}\,)\,) \tag{24}$$

which depends on the velocity of the particle. The similar structure antisymmetric in $\vec{v}$ and $\delta\vec{v}$ is present for any particle with nonzero angular momentum. This amplitude is remarkable since does not depend on the internal structure of the particle: it can be an elementary pointlike object, or a bound state. It depends only on its spin. Actual dynamics affects only the part of the amplitude symmetric in $\vec{v}$ and $\Delta\vec{v}$. The origin of the antisymmetric term is simply the transformation properties of the spin wavefunctions and roots to the noncommutativity of Lorentz boosts $U(\vec{v}\,)$ applied in different directions.

Let us illustrate this point. The state $|A(\vec{v}\,)\rangle$ can be obtained boosting the rest-frame particle, $U(\vec{v}\,)|A(0)\rangle$; likewise $|A(\vec{v}+\Delta\vec{v})\rangle = U(\vec{v}+\Delta\vec{v}\,)|A(0)\rangle$. Nontrivial dependence of the overlap $\langle A(\vec{v}+\Delta\vec{v}\,)|A(\vec{v}\,)\rangle$ on the frame velocity $\vec{v}$ is nothing but the fact that $U(\vec{v}+\Delta\vec{v}\,) \neq U(\Delta\vec{v})\cdot U(\vec{v}\,)$ in terms bilinear in both velocities. The commutator of the two infinitesimal boost operators is the rotation in the $(\Delta\vec{v},\vec{v}\,)$ plane; its action on a state amounts to the operator of angular momentum. Thus, the antisymmetric in $\vec{v}$ and $\Delta\vec{v}$ piece of the transition amplitude for a particle

directly measures its spin.[2] As is clear from the preceding discussion, the piece of the amplitude we are interested in comes from the phenomenon known as *Thomas precession*.

Using this general property of the transition amplitudes, we can imagine measuring separately the heavy quark spin and the total spin of $B^{(*)}$ meson in the following *gedanken* experiment which would bring us close to the sum rule. We start with the $B^{(*)}$ meson at rest and consider double interaction (scattering) of the external weak current on the b quark, Fig. 6. In the first act the hadron is accelerated to velocity $\vec{v}$, the second interaction additionally changes velocity by $\Delta\vec{v}$. Time $t_2 - t_1$ between the two interactions is at our disposal, and we can vary it. The energy variable ω conjugated to $t_2 - t_1$ actually measures the energy of the produced hadrons. For our purpose we project the final state onto $B^{(*)}$ and, again, select only the part of the amplitude antisymmetric in $\vec{v}$ and $\Delta\vec{v}$.

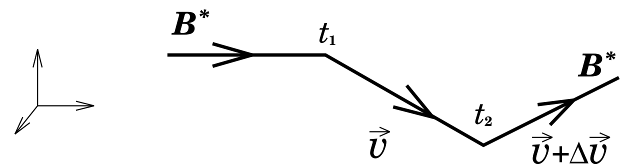

Figure 6: Double scattering process on heavy quark which measures total spin of B^* (at large $t_2 - t_1$) or spin of b quark (at $t_2 \to t_1$) via Thomas precession.

If we consider the elastic transition where the first act of interaction produces only moving $B^{(*)}$, the amplitude is given by the total spin of $B^{(*)}$. This changes if we add other intermediate states. If we sum the amplitude over *all* b hadrons appearing in the intermediate state, we actually measure only the spin of the heavy quark. Indeed, this totally inclusive amplitude corresponds to zero time separation $t_1 = t_2$, so that the light cloud surrounding heavy quark is decoupled having no time to follow up the scattering. These facts are explicit in the computation of both the elastic contribution to the scattering amplitude, and of its OPE expansion [1, 11]. Therefore, the contribution of the inelastic states alone directly measures *spin of light degrees of freedom*.

Directing the interested reader to the above original papers for the formal OPE derivation, here we sketch the quantum mechanical way to obtain the first sum rule. Our starting point is the expression for the SV transition amplitudes

$$\langle k(\vec{v})|J_0(0)|B^{(*)}(0)\rangle = -v_j \frac{\langle k|\pi_j|B^{(*)}\rangle}{\epsilon_k} \;, \tag{25}$$

and similarly for axial currents, up to spin-related factors. Here $\pi_j = iD_j$ is the momentum operator of the heavy quark. Below we also use the nonrelativistic

[2]Should we consider a vector current instead of the scalar one, there would be an additional term related to the Lorentz transformation of the current itself.

energy $\pi_0 = iD_0 - m_Q$. The r.h.s. in the above relation is written in usual quantum-mechanical notations. In the second-quantized form

$$\langle k(\vec{v}\,)|J_0(0)|B^{(*)}(0)\rangle = -v_j \frac{\langle k(\vec{v}=0)|\bar{Q}iD_jQ(0)|B^{(*)}(0)\rangle}{\epsilon_k} \;. \tag{26}$$

Relations for the SV transition amplitudes to the P-wave states are most easily obtained recalling that they are overlaps $\langle k(\vec{v}\,)|B(\vec{v}=0)\rangle$. Then we use the general rule

$$|H_Q(\vec{v}\,)\rangle \;=\; |H_Q(0)\rangle + \pi_0^{-1}\,(\vec{v}\vec{\pi}\,)\,|H_Q(0)\rangle \;+\; \mathcal{O}(\vec{v}^{\,2})\;. \tag{27}$$

which nicely elucidates the meaning of the small velocity sum rules: the operator $\pi_0^{-1}(\vec{v}\vec{\pi})$ acting on $|H_Q\rangle$ is the generator of the boost along the direction of $\vec{v}$. Indeed, to get $|H_Q(\vec{v}\,)\rangle$ one must find the eigenstate of the Hamiltonian with heavy quark moving with the momentum $\vec{q} = m_Q\vec{v}$. The only part which explicitly depends on momentum comes from the heavy quark Hamiltonian $\frac{\vec{\pi}^2}{2m_Q}$ (plus higher terms in $1/m_Q$). We use the relation $\exp\left(-i\vec{q}\vec{x}\,\right)\mathcal{H}_Q\, \exp\left(i\vec{q}\vec{x}\,\right) = \mathcal{H}_Q + \vec{v}\vec{\pi} + m_Q\vec{v}^{\,2}/2$ and drop the last term which is a constant; A_0 obviously commutes with x. Then Eq. (27) represents the first-order perturbation theory in $\delta\mathcal{H} = \vec{v}\vec{\pi}$, where $-\pi_0$ plays the role of the unperturbed Hamiltonian $\mathcal{H}_0$ (further details can be found in Ref. 4 , Eq. (178) and Sect. VI). In the second-quantized notations the same relation takes the form

$$|H_Q(\vec{v}\,)\rangle \;=\; |H_Q(0)\rangle \;+\; \int \mathrm{d}^3\vec{x}\;\bar{Q}\,\pi_0^{-1}\,(\vec{v}\vec{\pi}\,)\,Q(x)|H_Q(0)\rangle \;+\; \mathcal{O}(\vec{v}^{\,2})\;. \tag{28}$$

The first sum rule (12), therefore requires computing the antisymmetric in j,k structure in the following sum

$$\sum_n \frac{\langle B^*|\pi_j|n\rangle\,\langle n|\pi_k|B^*\rangle}{(E_n - E_{B^*})^2}\;. \tag{29}$$

The energies E_n, E_{B^*} are eigenvalues of the total Hamiltonian

$$\begin{aligned}\mathcal{H} \;&=\; \mathcal{H}_{\rm light} + \mathcal{H}_Q \qquad\qquad (30)\\ &= \mathcal{H}_{\rm light} \,-\, A_0 + \frac{1}{2m_Q}\,(\vec{\pi}^2 + \vec{\sigma}\vec{B}) + \frac{1}{8m_Q^2}\left[-(\vec{D}\vec{E}) + \vec{\sigma}\cdot\{\vec{E}\times\vec{\pi} - \vec{\pi}\times\vec{E}\}\right] + \mathcal{O}(1/m_Q^3),\end{aligned}$$

where the $1/m_Q^2$ piece is the sum of the Darwin and convection current (LS) terms. In order to compute the sum we, following usual nonrelativistic quantum mechanics, use the commutation relation expressing $\vec{\pi}$ as the commutator of the heavy quark coordinate $\vec{x}$ with the Hamiltonian, but include the $1/m_Q^2$ terms:

$$\pi_j = -i\,m_Q\,[x_j, \mathcal{H}] \;+\; \frac{i}{8m_Q}\left(E_j + 2\,i\left[\vec{\sigma}\times\vec{E}\right]_j\right)\;. \tag{31}$$

Since each commutator with $\mathcal{H}$ kills one factor $E_n - E_{B^*}$ in the denominator, the leading in m_Q term is given by $m_Q^2\left(\langle B^*|x_jx_k|B^*\rangle - \langle B^*|x_j|B^*\rangle\,\langle B^*|x_k|B^*\rangle\right)$ which

is symmetric over j and k. The antisymmetric part comes in the next order. To compute it we express, to the leading order in m_Q, chromoelectric field as the commutator of the heavy quark momentum and the Hamiltonian, $E_l = i[\pi_l, \mathcal{H}]$. Then we finally get

$$\sum_n \frac{\langle B^*|\pi_j|n\rangle\,\langle n|\pi_k|B^*\rangle}{(E_n - E_{B^*})^2} = m_Q^2\,[\langle B^*|x_j x_k|B^*\rangle - \langle x_j\rangle\,\langle x_k\rangle] \\ + \tfrac{i}{8}\,[\langle B^*|\pi_j x_k + x_j \pi_k|B^*\rangle - \langle \pi_j\rangle\langle x_k\rangle - \langle \pi_k\rangle\langle x_j\rangle] \\ + \tfrac{1}{4}\,[\langle B^*|[\vec\sigma\times\vec\pi]_j x_k + x_j[\vec\sigma\times\vec\pi]_k|B^*\rangle - \langle[\vec\sigma\times\vec\pi]_j\rangle\langle x_k\rangle - \langle[\vec\sigma\times\vec\pi]_k\rangle\langle x_j\rangle]\;. \tag{32}$$

(The terms with separate expectation values of $\vec\pi$ and $\vec x$ can be discarded.) The basic commutator relation $[x_j, \pi_k] = i\delta_{jk}$ thus yields for the antisymmetric part the matrix element of the heavy quark spin, $\frac{i}{4}\,\epsilon_{jkl}\,\sigma_l$. It comes from the LS term in the heavy quark Hamiltonian. This matrix element is given by $\frac{1}{4}\,(\varepsilon_j'^*\varepsilon_k - \varepsilon_k'^*\varepsilon_j)$, where ε refer to the polarization vectors of B^*. On the other hand, the explicit summation over the members of the hyperfine P-wave multiplets and their polarizations yields for the antisymmetric part the same spin structure $(\varepsilon_j'^*\varepsilon_k - \varepsilon_k'^*\varepsilon_j)$ multiplied by $-\tau_{3/2}^2$ and $\tau_{1/2}^2$ for the $\frac{3}{2}$- and $\frac{1}{2}$-states, respectively (see, e.g. Ref. [1], Sect.4.1). In this way the general structure of the sum rule (12) is reproduced.

A subtlety must be noted at this point which was previously discussed in the similar context in Ref. [17]. In order to perform the summation over intermediate states, we had to include in Eq. (32) all possible intermediate heavy quark states $|n\rangle$ which have nonvanishing matrix elements. They can be labeled by their explicit excitation number and the total momentum (not indicated explicitly). As mentioned above, we consider the zero-momentum matrix elements, so that the summation over overall momentum is removed, cf. Eq. (28). This does not literally apply, however, to the diagonal transition into the same $B^{(*)}$ states. Although the matrix elements naively contain δ-function of momentum, the energy denominator in this case amounts to $(\vec p^{\,2}/2m_Q)^2$ yielding singularity at small $\vec p$. This singularity must be treated properly and yields a finite contribution. It must be subtracted from the sum, since the sum rules include only the transitions into the excited states.

The diagonal contribution can be computed without performing actual discretization of the problem, employing the trick used in Ref. [17]. Considering the infinitesimal momenta of $B^{(*)}$ we can relate the matrix elements of the b quark momentum $\vec\pi$ to the matrix elements of the total hadron momentum $\vec P$:

$$\langle B^{(*)}(\vec p)|\pi_j|B^{(*)}(0)\rangle = \frac{m_Q}{M_{B^{(*)}}}\langle B^{(*)}(\vec p)|P_j|B^{(*)}(0)\rangle\;. \tag{33}$$

In the present problem we can neglect the deviation of ratio $m_Q/M_{B^{(*)}}$ from unity. We then need to evaluate the following sum over all momenta of $B^{(*)}$:

$$\int \frac{\mathrm{d}^3\vec p}{(2\pi)^3}\,\frac{\langle B^*(0)|P_j|B^*(\vec p)\rangle\langle B^*(\vec p)|P_k|B^*(0)\rangle}{(E_{B^*}(\vec p) - M_{B^*})^2}\;; \tag{34}$$

This, however, is nothing but the general sum rule (32) applied to B^* mesons as (almost) free elementary particles. (The nontrivial answer emerges due to *ad hoc* interaction assumed to render the spectrum discrete.) The antisymmetric part here amounts then to the expectation value of the spin of B^* as a whole, and is given by the same polarization structure as above, but with the additional factor of 2. Subtracting this contribution from Eq. (32) finally yields the spin sum rule (12).

The quantum mechanical derivation of the second spin sum rule (14) is even simpler if we recall the expressions (27), (28) for the moving heavy hadron state. On then has

$$\langle B^*(\vec{v})|\pi_j|B^*(0)\rangle = v_k\langle B^*(0)|\pi_k\pi_0^{-1}\pi_j|B^*(0)\rangle = -v_k{\sum_n}' \langle B^*|\pi_k\pi_0^{-1}|n\rangle\epsilon_n\langle n|\pi_0^{-1}\pi_j|B^*\rangle, \tag{35}$$

where all quantum-mechanical states have zero momentum, and the prime indicates that summation does not include the diagonal transition. In writing this we have used the fact that the operator $-\pi_0$ plays the role of the Hamiltonian for the states with zero momentum. Indeed, due to the heavy quark equation of motion $\pi_0 Q(x)=0$ the relation holds for an arbitrary operator O

$$\langle n|\bar{Q}\pi_0 O Q(0)|m\rangle = (E_m - E_n)\langle n|\bar{Q}OQ(0)|m\rangle \ .$$

The transition amplitudes in the sum in Eq. (35) are directly related to τ's. As before, the contributions to the symmetric and antisymmetric in j and k structures there are given by $2\tau_{3/2}^2+\tau_{1/2}^2$ and $\tau_{3/2}^2-\tau_{1/2}^2$, respectively, which yields Eq. (22) with $\overline{\Lambda}$ and $\overline{\Sigma}$ given by sum rules (13), (14).

The spin sum rule (12) provides the rationale for the experimental fact that vector mesons B^*, D^* are heavier than their hyperfine pseudoscalar partners B, D. Indeed, if the sum rule for μ_G^2 is dominated by the low-lying states then μ_G^2 must be of the same sign as the constant in Eq. (12), which dictates the negative energy of the heavy quark spin interaction in B and positive in B^*.

The sum rules (11)–(18) obviously entail a set of exact QCD inequalities. They are similar to those which have been with us since the early 1980's [18] and reflect the most general features of QCD (such as the vector-like nature of the quark-gluon interaction). The advent of the heavy quark theory paved the way to a totally new class of inequalities among the fundamental parameters. As with the old ones, they are based on the equations of motion of QCD and certain positivity properties. All technical details of the derivation are different, however, as well as the sphere of applications. The first in the series is the Bjorken inequality $\varrho^2 \geq \frac{1}{4}$ [10]. We, in fact, have a stronger *dynamical* bound $\varrho^2 \geq \frac{3}{4}$ following from the sum rule (12). It ensures that at least some of the inelastic amplitudes must be nonzero; therefore, the bound state with nonzero spin of light cloud cannot be structureless (pointlike) regardless of bound-state dynamics. The Bjorken bound is not dynamical in this respect: the Isgur-Wise function $\xi(vv')=\sqrt{2/(1+vv')}$ with $\varrho^2=\frac{1}{4}$ is the one for the structureless particle. Comparison to the QCD sum rule evaluation $\varrho^2 = 0.7\pm 0.1$

[19] suggests that the new bound can be nearly saturated. We also have the bound $\overline{\Lambda} > 2\overline{\Sigma}$. Other bounds include $\mu_\pi^2 \geq \mu_G^2$ and $\rho_D^3 \geq |\rho_{LS}^3|/2\,,\ \rho_D^3 \geq -\rho_{LS}^3\,$. We also have direct inequalities between the parameters of different dimensions,

$$\mu_\pi^2 \geq \frac{3\overline{\Lambda}^2}{4\varrho^2-1}\;, \qquad \rho_D^3 \geq \frac{3}{8}\frac{\overline{\Lambda}^3}{(\varrho^2-1/4)^2}\;, \qquad \rho_D^3 \geq \frac{(\mu_\pi^2)^{3/2}}{\sqrt{3(\varrho^2-1/4)}}\;. \tag{36}$$

All these inequalities are saturated provided that only one excited state contributes.

The old nonrelativistic quark models often do not respect the general relations discussed above. They typically yield small difference between the $\frac{1}{2}$- and $\frac{3}{2}$- P-wave states, and therefore hardly can be viewed reliable. A class of relativistic models was developed recently [20] which incorporate proper Lorentz boost transformations and obey the exact commutation relations. Consequently, they naturally predict suppression of the $\tau_{1/2}$ amplitudes compared to $\tau_{3/2}$. Likewise, they obey the bound $\varrho^2 > \frac{3}{4}$. It seems appealing to reexamine the quark model predictions for processes with heavy flavors in the framework of the new models [20].

3.1 Hard QCD and normalization point dependence

The sum rules (11)–(18) express the heavy quark parameters, including $\overline{\Lambda} = M_B - m_b$, μ_π^2 and μ_G^2, as the sum of observable quantities, products of the hadron mass differences and the transition probabilities. The observable quantities are scale independent. How then, say, $\overline{\Lambda} = M_B - m_b$, μ_π^2 and μ_G^2 happen to be μ-dependent?

The answer is that in quantum field theory such as QCD the sums over excited states are generally ultraviolet divergent due to physics at $E \gg \Lambda_{\rm QCD}$. In contrast to ordinary quantum mechanics they are not saturated by a few lowest states with contributions fading away fast in magnitude with the excitation number. The contributions of hadronic states with $\epsilon_k \gg \Lambda_{\rm QCD}$ are dual to what we calculate in perturbation theory using its basic objects, quarks and gluons. The latter yield the continuous spectrum and can be evaluated perturbatively using isolated quasifree heavy quarks as the initial state. The final states are heavy quarks plus a certain number of gluons and light quarks. It is the difference between the actual hadronic and quark-gluon transitions that resides at low excitation energies.

Therefore, in order to make the sum rules meaningful, we must cut off the sums at some energy μ which then makes the expectation values μ-dependent. The simplest way is merely to extend the sum only up to $\epsilon_k < \mu$. This is the convention we normally use. Thus, in actuality all the sums in the relations (11)–(18) must include the condition $\epsilon_k < \mu$, which we omitted there for the sake of simplicity, and all the heavy quark parameters are normalized at the scale μ. The exception is the superconvergent spin-nonsinglet sum rules (12) and (14) where in the perturbative domain $\mu \gg \Lambda_{\rm QCD}$ such μ-dependence is power suppressed by factors $\alpha_s \Lambda_{\rm QCD}/\mu$ and can be neglected.

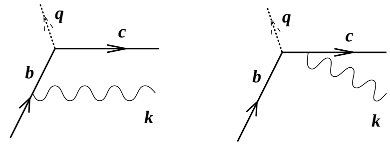

Figure 7: Perturbative diagrams determining the high-energy asymptotics of the heavy quark transition amplitudes and renormalization of the local operators.

The high-energy tail of the transitions, to the order α_s, is given by the quark diagrams in Figs. 7 with

$$2\sum_m ... + \sum_n ... \;\to\; \int \frac{\mathrm{d}^3\vec{k}}{2\omega}$$

where $(\omega, \vec{k})$ is the momentum of the real gluon. The spin-singlet amplitudes are just a constant proportional to g_s. Performing simple calculations we arrive at the first-order term in the evolution of, say, $\mu_\pi^2(\mu)$ [17],

$$\frac{\mathrm{d}\mu_\pi^2(\mu)}{\mathrm{d}\mu^2} \;=\; \frac{4}{3}\,\frac{\alpha_s}{\pi} + \ldots\ . \tag{37}$$

Purely perturbatively, the continuum analogs of $\tau_{1/2}$ and $\tau_{3/2}$ are equal, and a similar additive renormalization of μ_G^2 and ρ_{LS}^3 is absent.

The perturbatively obtained evolution equation (37) and the similar one for the chromomagnetic operator stating its anomalous dimension $-3\alpha_s/2\pi$ allow one to determine asymptotics of $\tau_{1/2}$ and $\tau_{3/2}$ at $\epsilon \gg \Lambda_{\rm QCD}$,

$$2\sum_m \epsilon_m^2 |\tau_{3/2}^{(m)}|^2 + \sum_n \epsilon_n^2 |\tau_{1/2}^{(n)}|^2 \;\;\to\;\; \frac{8\alpha_s(\epsilon)}{9\pi}\,\epsilon\,\mathrm{d}\epsilon\ , \tag{38}$$

$$\sum_m \epsilon_m^2 |\tau_{3/2}^{(m)}|^2 - \sum_n \epsilon_n^2 |\tau_{1/2}^{(n)}|^2 \;\;\to\;\; -\frac{3\alpha_s(\epsilon)}{2\pi}\,\frac{\mathrm{d}\epsilon}{\epsilon}\left[\sum_{\epsilon_m<\epsilon} \epsilon_m^2 |\tau_{3/2}^{(m)}|^2 - \sum_{\epsilon_n<\epsilon} \epsilon_n^2 |\tau_{1/2}^{(n)}|^2\right]. \tag{39}$$

The last bracket is $\frac{1}{6}\mu_G^2(\epsilon)$. Equation (38) can be extended to higher orders in α_s, this amounts to using the so-called dipole coupling $\alpha_s^{(d)}(\epsilon)$ introduced in Ref. [8]:

$$\alpha_s^{(d)}(\epsilon) = \bar{\alpha}_s\left(\mathrm{e}^{-5/3+\ln 2}\epsilon\right) - 3\left(\tfrac{\pi^2}{6} - \tfrac{13}{12}\right)\tfrac{\alpha_s^2}{\pi} \,+\, \mathcal{O}(\alpha_s^3)\ . \tag{40}$$

($\bar{\alpha}_s$ is the standard $\overline{\rm MS}$ strong coupling). Therefore we get a number of exact perturbative evolution equations [8]

$$\mu\frac{\mathrm{d}\varrho^2(\mu)}{\mathrm{d}\mu} \;=\; \frac{8}{9}\,\frac{\alpha_s^{(d)}(\mu)}{\pi}\,, \tag{41}$$

$$\frac{\mathrm{d}\overline{\Lambda}(\mu)}{\mathrm{d}\mu} \;=\; \frac{16}{9}\,\frac{\alpha_s^{(d)}(\mu)}{\pi}\,, \tag{42}$$

$$\frac{\mathrm{d}\mu_\pi^2(\mu)}{\mathrm{d}\mu} \;=\; \frac{8}{3}\,\frac{\alpha_s^{(d)}(\mu)}{\pi}\,\mu\,. \tag{43}$$

Using Eq. (39) we can estimate the contribution of the high-energy states in the sum rules (12) and (14):

$$\sum_{\epsilon_m<\mu} |\tau_{3/2}^{(m)}|^2 - \sum_{\epsilon_n<\mu} |\tau_{1/2}^{(n)}|^2 \simeq \frac{1}{4} + \frac{\alpha_s(\mu)}{8\pi}\,\frac{\mu_G^2(\mu)}{\mu^2}\,, \tag{44}$$

$$\sum_{\epsilon_m<\mu} \epsilon_m |\tau_{3/2}^{(m)}|^2 - \sum_{\epsilon_n<\mu} \epsilon_n |\tau_{1/2}^{(n)}|^2 \simeq \frac{\overline{\Sigma}}{2} + \frac{\alpha_s(\mu)}{4\pi}\,\frac{\mu_G^2(\mu)}{\mu}\,; \tag{45}$$

they are power suppressed and presumably small in the perturbative domain.

3.2 On the saturation of the sum rules

The question of the saturation of the heavy quark sum rules (in particular, the lower ones (11)–(16)) is of primary importance for phenomenology of the heavy quark expansion. The sum rules at large enough normalization point μ tell us what the asymptotic value on the right-hand side is, and perturbation theory tells us its μ dependence. It is a dynamical question starting from which scale μ_0 this behavior applies. For superconvergent sum rules (12) and (14) this is the question at which scale the sums approach the stated values with a reasonable accuracy. In order to sensibly apply quantitative $1/m_Q$ expansion, one must have $m_Q>\mu_0$, presumably, $m_Q\gg\mu_0$. While this is, probably, the case for b particles, such a hierarchy is not obvious *a priori* in charm.

The existing numerical evaluations of $\overline{\Lambda}$ and μ_π^2 at the scale near $1\,\mathrm{GeV}$ suggest rather large values, approximately $0.7\,\mathrm{GeV}$ and $0.6\,\mathrm{GeV}^2$, respectively, which impose rather tight constraints. These facts were often neglected under various pretexts, including challenging the accuracy of the QCD sum rules evaluations of μ_π^2.

Nevertheless, certain constraints following from the sum rules are tight and robust simultaneously. Namely, the value of $\mu_G^2 \simeq 0.4\,\mathrm{GeV}^2$, as extracted almost directly from $B^{(*)}$ and $D^{(*)}$ masses, has hardly been challenged. By virtue of the sum rules, the value of μ_π^2 is at least as large. Thus, regardless of the accuracy in evaluations of the kinetic expectation value, the question can be phrased in terms of the generally accepted value of μ_G^2. At which minimal scale μ_0 the value of $\mu_G^2(\mu_0)$ reaches 0.3 or $0.4\,\mathrm{GeV}^2$? If this scale is below $1\,\mathrm{GeV}$, large $\overline{\Lambda}$ and μ_π^2 are almost inevitable. If, however, $\mu_G^2(1\,\mathrm{GeV})$ is significantly below $0.4\,\mathrm{GeV}^2$, the chances for success in $1/m_Q$ expansion in charm are slim.

Similar constraints follow from the sum rule (14) and, in particular, (12). If these sum rules are saturated below $1\,\mathrm{GeV}$, we expect large $\overline{\Lambda}$ and μ_π^2. If the saturation scale is higher, perturbative treatment of the scales $\sim m_c$ only slightly exceeding $1\,\mathrm{GeV}$ is not justified.

The estimates for $\tau_{3/2}$ and $\tau_{1/2}$ for the lowest P wave states group around 0.4, with $\epsilon_{3/2}^{(1)} \gtrsim \epsilon_{1/2}^{(1)} \simeq 400$ to $500\,\mathrm{MeV}$ (for the review see Ref. [21]). The old quark models yielded close values for $\frac{3}{2}$- and $\frac{1}{2}$-states. The more recent relativistic models [20] predict noticeable suppression of the transitions into $\frac{1}{2}$-states compared to the

$\frac{3}{2}$-amplitudes. Similar absolute values were reportedly extracted for $\tau_{3/2}$ from the overall experimental yield of the corresponding charmed P wave states [22]. It is evident that such transitions amplitudes fall short of saturating the sum rules,

$$\delta^{(1)}_{3/2}\,\mu_G^2 \simeq 0.2\,\mathrm{GeV}^2\,, \qquad \delta^{(1)}_{3/2}\,\overline{\Lambda} \simeq 0.3\,\mathrm{GeV}\,, \qquad \delta^{(1)}_{3/2}\,S_{\rm light} \simeq 0.3$$

($S_{\rm light}$ denotes the sum in Eq. (12) for the light cloud spin). For the spin-dependent sum rules, the first and the third entries, the $\frac{1}{2}$-states would further decrease the values. In principle, the lowest states alone should not necessarily saturate the sum rules, even though the idea of the dominance of the lowest state contributions is very appealing. Let us mention that in the 't Hooft model all heavy quark sum rules are saturated with amazing accuracy by the first excitations [23]. Is such a possibility excluded in QCD?

Probably, not completely. The dominance of the first excitation with $\epsilon \simeq 500\,\mathrm{MeV}$ (recall that one must use the asymptotic $m_Q \to \infty$ values of the excitation energies and amplitudes) is still possible if the QCD sum rules underestimate the value of $\tau^{(1)}_{3/2}$.[3] Experimental determinations of τ's are also questionable since $1/m_c$ corrections are not accounted for there. The estimates in the 't Hooft model suggest that they can be very large. In the cases where they are known explicitly in QCD, the $1/m_c$ terms generally turn out to be very significant as well [24].

Another – and, apparently, the most natural – option is that there are new states with the masses around 700 MeV with similar, or even larger $\tau^{(2)}_{3/2} \simeq 0.4$ to 0.5; they can be broad and not identified with clear-cut resonances. The $\frac{1}{2}$-states must be yet depleted up to this scale. All such states can be produced in semileptonic b decays and observed as populating the domain of hadronic invariant mass below or around 3 GeV. It will be important to explore these questions in experiment.

The branching fraction of experimentally identified narrow $j=\frac{3}{2}$ P-wave states does not exceed a percent level. A larger fraction was reported recently for the broad hadronic distributions in $B \to D^{(*)}\pi\,\ell\nu$ decays [25], between 2% and 3.5%. Such an yield helps to fill the gap between the total semileptonic fraction $BR_{\rm sl} \simeq 10.5\%$ and identified exclusive channels. Since the observed $D^{(*)}\pi$ mass distribution is very broad, these states were attributed to the $j=\frac{1}{2}$ resonances which are expected to have large decay width. If this is correct, we would rather observe significant negative contributions to the sum rules (12) and (16) from the domain of $\epsilon \lesssim 700\,\mathrm{MeV}$. This possibility does not look natural from the theoretical viewpoint; neither it is supported by the predictions of the recent relativistic quark models [20, 16]. If it is nevertheless realized in reality, we would have to expect quite large values of $\overline{\Lambda}$ and, in particular, μ_π^2.

We note, however, that there is an alternative interpretation of the data: the reported enhanced yield of broadly distributed $D^{(*)}\pi$ can be the manifestation of

[3] Let us note that the technology of the QCD sum rules assumes the approximate duality starting $\epsilon = 1\,\mathrm{GeV}$ or even lower (the energies are counted from the heavy quark mass there). Therefore, accepting poor saturation of the exact heavy quark sum rules at this scale and simultaneously relying on the QCD sum rules predictions is not selfconsistent.

nonresonant (continuum) production; then it can belong to the $\frac{3}{2}$-states. If this is the case, one would get a consistent picture of conventional saturation of the heavy quark sum rules at the usual energy scale under 1 GeV, without necessity to invoke new paradigms like a too high onset of quark-hadron duality.

The continuum yield is routinely assumed to be small compared to the resonant contribution. This, however, is more the heredity of naive quark models than the fact based directly on QCD. The absolute branching fraction of this yield is rather small, about 20% of the overall semileptonic rate, and agrees well with the general fact that nonresonant contributions are $1/N_c$-suppressed. It seems certain that such a possibility must be carefully explored before alarming conclusions are drawn.

Interpreting the data mentioned above can be obscured by the potentially significant $1/m_c$ effects. The spin of light degrees of freedom itself is a well defined quantum number only as long as the limit $m_c \to \infty$ is considered. While deviations from this academic case are possibly under control for D and D^*, the situation can be much worse for the excited states, in particular, in the mass range we discuss. Here the energy of the light cloud already exceeds m_c itself. The new relativistic quark models [20] can gives us a sense of possible significance of such effects.

4 $\mathrm{BR}_{\rm sl}(B)$ and n_c

Before concluding, I would like to mention another nonstandard possibility which can affect the analysis of $\mathrm{BR}_{\rm sl}(B)$ and the average number of charm quarks per B decay n_c. As explained elsewhere, the problem of semileptonic fraction is usually considered in conjunction with n_c – the latter constrains the total decay rate in the $b \to c\bar{c}\,s(d)$ channel and thereby allows to isolate potential uncertainties in the latter due to limited energy release (see, e.g., Ref. [1], Sect. 7.1).

The point is that there is a certain assumption in the standard analysis which goes beyond applying the OPE *per se*. Namely, the partial decay rate mediated by quark transition $b \to c\bar{c}s$ and computed via the OPE, is implicitly equated to the probability of observing hidden charm, or open charm plus anticharm states in the final state. Stating differently, the charmless final states and those having a single charm particle are presumed to be totally independent of the $b \to c\bar{c}s$ quark level transitions. While this is certainly the case for the single-charm channels, it is an additional assumption for the charmless decays; the possibility that it is violated was considered in Refs. [27].

Since charm quark is heavy in the scale of strong interactions, it is more than plausible that this assumption is accurate enough. Yet it must be realized that we do not derive it from the first principles of QCD. In particular, the OPE does *not* tell us this.[4] The OPE predicts the total decay probability induced by the particular term in the weak Lagrangian, $\bar{c}\gamma_\mu(1-\gamma_5)b\;\bar{s}\gamma_\mu(1-\gamma_5)c$, but it cannot say if it is

[4]More precisely, I am not aware of the way to justify it from the OPE *per se*, without recourse to additional assumptions.

exhausted by the final states with two c quarks, or it is shared with the charmless final states. The OPE width must include all rescattering processes possible in the final state. Moreover, it is known that failure to include such contributions can even lead to violation of general theorem like absence of $\Lambda_{\rm QCD}/m_Q$ corrections to the decay widths [26]. It is worth emphasizing that we do not mean the usual effects from the standard Penguin operators – these properly describe the corrections to the *total* decay rates. Our point here is the effects of final states interactions which reshuffle the decay rate between different “apparent quark channels”, but do not change the overall rate.

Since charm quark is relatively heavy, the rescattering probability between $c\bar{c}$ and light $q\bar{q}$ (or gluonic) states must be suppressed. Yet even the relatively small fraction can be significant here. We can introduce the phenomenological parameter $\mathcal{C}$ to measure such a probability:

$$\mathcal{C} = \frac{\Gamma_{b\to c\bar{c}s}(B \to \text{charmless})}{\Gamma^{\rm tot}_{b\to c\bar{c}s}}\;, \tag{46}$$

where both widths are understood as generated by the the $(\bar{c}b)(\bar{s}c)$ or $(\bar{c}b)(\bar{d}c)$ Lagrangians. (We assume they are normalized at the scale around m_b to isolate conventional Penguin operators). By construction, $\mathcal{C} \neq 0$ does not change the total nonleptonic width, but it does affect n_c:

$$n_c \to n_c - 2\,\mathcal{C}\!\cdot\!{\rm BR}^{\rm OPE}(B \to c\bar{c}\,X)\;. \tag{47}$$

Strictly speaking, $\mathcal{C}\!\neq\!0$ would mandate existence of the opposite processes as well, say, double-charm events in the decays mediated by $b \to u\bar{u}d$ transitions, beyond the contributions of the Penguin operators. However, the strong suppression of the KM disfavored channels makes such effects insignificant.

There exists an experimental bound on such rescattering processes: ${\rm BR}(B \to$ no charm$) \lessapprox 4\%$ [28] leads to $\mathcal{C} \lessapprox 0.2$. Even being conservative, we would expect $\mathcal{C}$ smaller than that. (The estimated ${\rm BR}^{\rm OPE}(B \to$ no charm) is below 1%.) But the message is clear: Any value of $\mathcal{P}\!=\!{\rm BR}^{\rm FSI}(B \to$ no charm) from such final state rescattering would decrease n_c by $2\,\mathcal{P}$. Therefore, even $\mathcal{P}$ about 3% would have the dramatic impact on the allowed domain of theoretical predictions in the $({\rm BR}_{\rm sl}, n_c)$ space. It therefore seems important to improve the direct experimental bound on ${\rm BR}(B \to$ no charm) down to 1 to 2%.

Acknowledgments: It is a pleasure to thank the organizers of the Durham Workshop and, in particular, Mike Whalley and Adrian Signer for hospitality and enjoyable atmosphere. I benefited from discussions with many participants, both theorist and experimentalists. Collaboration with I. Bigi, M. Eides, M. Shifman and A. Vainshtein is gratefully acknowledged. I am pleased to thank the authors of Ref. [16], especially A. Le Yaouanc and L. Oliver for interesting exchanges regarding spin sum rules and quark models. This work was supported in part by NSF under grant number PHY96-05080 and by RFFI under grant No. 99-02-18355.

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